Precoding-Based Waveforms for 5G New Radios Using GFDM Matrices

Introduction
Orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA) have been applied in many modern communication systems because of several advantages including eliminating inter-symbol interference caused by multipath propagation with low-complexity implementation. In particular, 4G LTE has used OFDMA as the basis of the physical-layer frame structure. However, OFDM/OFDMA suffers from some well-known drawbacks such as sensitivity to frequency offset and high peak-to-average power ratio which require accurate synchronization and highly linear power amplifiers, respectively, to work.

In 5G, demands arising from low-latency applications, energy efficiency, support of multiple numerologies, relaxation of synchronization, etc., impose constraints on a transmission waveform such as low out-of-band emission (OOBE) and low peak-to-average power ratio (PAPR) on which OFDM/OFDMA generally does not perform well. Several waveforms alternative to OFDM have been proposed and studied, including filter-band multicarrier (FBMC), universal-filtered multicarrier (UFMC), generalized frequency division multiplexing (GFDM), etc. Many of these waveforms call for advantages such as spectral containment and power amplifier efficiency and have been proposed for 3GPP’s consideration as the waveforms for New Radio (NR).

Nevertheless, in 2016, the 3GPP decided to use OFDM-based waveforms for NR partly because of complexity and compatibility. This essentially excludes many of the aforementioned alternative waveforms. However, the agreement does allow some additional processing on top of OFDMA,
such as additional filtering, windowing, and precoding, so that there are still rooms for improvement in terms of waveform OOB, PAPR, and other desirable properties. A typical block diagram is shown in Figure 1. Filtering-based waveforms have advantages in OOB (e.g., filtered-OFDM, f-OFDM), but they have bad performance in PAPR. Windowing-based waveforms (such as windowed-OFDM, weighted overlap-and-add OFDM, WOLA-OFDM) have generally less issue on PAPR and complexity. However, it may cause detection performance degradation due to symbol extension.

Precoding-Based Waveforms

In this article, we focus on precoding based waveforms since they generally do not impose additional CP budget that may lead to possible inter-symbol interference, compared with filtering-based and windowing-based waveforms. A linear precoder is characterized by an $S \times S$ matrix $P$ which performs a matrix multiplication on the data vector $s[n]$ to obtain precoded vector $u[n] = P \, d[n]$. One promising representative example of precoding-based waveform is discrete Fourier transform spread OFDM (DFT-S-OFDM), where the precoding matrix is chosen as $P = W_S$, where $W_S$ is the normalized DFT matrix whose $(k, l)$-entry is $[W_S]_{kl} = e^{j2\pi kl/S}/\sqrt{S}$. A number of variants of DFT-S-OFDM have been proposed in 3GPP for NR. For example, zero tail (ZT) DFT-S-OFDM, guard interval (GI) DFT-S-OFDM, and unique-word (UW) DFT-S-OFDM.

In the most general form, a precoding-based waveform is completed determined by the $S \times S$ precoding matrix $P$. So, the waveform can be designed to meet system requirements on OOB or PAPR with the degree of freedom being $S^2$ complex-valued coefficients, a number much higher than that for filtering- or windowing-based counterparts. However, a precoder with arbitrarily chosen entries has a high implementation complexity (at the order of $O(S^3)$ for the transmitter and $O(S^3)$ for the receiver) and is not desirable. If we impose some constraints on the precoder’s structure, a low-complexity implementation (e.g., at the order of $O(S \log S)$) may exist. The DFT-S-OFDM waveform satisfies this criterion since its precoding matrix, $P = W_S$, is an DFT matrix and has well-known fast implementation. However, by choosing $P = W_S$, there is no more degrees of freedom for waveform design.

Using GFDM Matrices in Precoding-Based Waveforms

We investigated a class of precoding-based waveform that has low-complexity transceiver implementations and still has $S$ design coefficients. Specifically, we choose the precoder to be $P = W_S A$ where $A$ is the GFDM matrix to be defined later. By choosing the precoder in this way, it is found that advantages of OOB can be inherited from GFDM. In addition, unlike GFDM which may not be compatible with OFDMA-based waveforms, the precoding-based waveform using GFDM matrices can naturally coexist with other users using conventional OFDMA. The GFDM matrix $A$ is determined by a prototype filter $g \in \mathbb{C}^S$ and two positive integers $K$ and $M$ such that $S = KM$. The integers $K$ and $M$ are referred to as the numbers of subcarriers and subsymbols, respectively, in the GFDM notations. For each $k = 0, 1, ..., K-1$ and $m = 0, 1, ..., M-1$, the vector $g_{k,m}$ is defined as the translated and modulated version of the prototype filter $g$: $[g_{k,m}]_n = [g]_{(n-mK \text{ mod } S)} e^{j2\pi kn/K}$. Then the GFDM matrix can be expressed as

$$A = \begin{bmatrix} g_{0,0} & \cdots & g_{0,K-1} & \cdots & g_{K-1,0} & \cdots & g_{K-1,K-1} & \cdots & g_{K-1,M-1} \end{bmatrix} \quad (1)$$

The GFDM matrix defined in Eq. (1) is central in implementation of GFDM systems. In a GFDM
system, only one GFDM matrix $\mathbf{A}$ as in (1) is used at the transmitter, which processes and arranges user data into resource elements composed of $K$ subcarriers times $M$ subsymbols. However, since GFDM subcarriers (and subsymbols) are not necessarily orthogonal to each other (unlike the case in OFDM systems), it is still not clear how “GFDMA”, i.e., multiple access based on using different GFDM subcarriers for different users, can be done without any interference. Our proposal of using GFDM matrices in precoding-based waveforms, fortunately, does not suffer from this issue since the multiple access is still based on OFDMA (that is, users are occupying different OFDM subcarriers which are orthogonal to each other) and each user can be associated with a GFDM-based precoder (not necessarily identical) without any concern to interfere with others. In fact, other users can even use non-GFDM precoders without any incompatibility. Therefore, the proposal allows coexistence of legacy OFDMA users and NR users, especially for multiple numerology and sub-6 GHz. For detailed explanations, readers may refer to [1].

**Resolving the GFDM Issues on Noise Enhancement and Receiver Complexity**

There are some more known drawbacks on GFDM that might have prevented it from being widely accepted so far. Now that we adopted GFDM matrices in our precoder design, we shall take a closer look on these issues since we might be inheriting these drawbacks as well [4]. First of all, it was noted that at least one of the parameters $M$ and $K$ shall be chosen as an odd integer, for otherwise the matrix $\mathbf{A}$ would become singular. Secondly, even when the matrix $\mathbf{A}$ is not singular, $\mathbf{A}$ is usually not unitary, resulting in noise amplification at the receiver when an inverse operation of the precoding is applied. Finally, it was argued that a GFDM receiver could be quite complicated unless the prototype filter $\mathbf{g}$ is limited to a small set of choices.

Fortunately, in our recent investigation [2], it is found that the aforementioned problems of GFDM can be mostly resolved by identifying a class of unitary GFDM precoders which have a noise enhancement factor as low as unity and a low transceiver complexity. It is proven in [2] that the matrix $\mathbf{A}$ can be expressed as

$$\mathbf{A} = (\mathbf{W}_M^H \otimes \mathbf{I}_K) \text{diag}(\text{vec}(\mathbf{G})) (\mathbf{W}_M \otimes \mathbf{W}_K^H)$$

where $\mathbf{G} = \sqrt{S} \text{reshape}(\mathbf{g}, K, M) \mathbf{W}_M$ is a $K \times M$ matrix derived from an affine transform of the prototype filter $\mathbf{g}$. From Eq. (2), it can be readily verified that $\mathbf{A}$ is a unitary matrix if and only if each entry of $\mathbf{G}$ is chosen as a complex number on the unit circle. A prototype filter $\mathbf{g}$ that corresponds to such a choice is referred to as a constant-modulus characteristic matrix (CMCM) filter. With a CMCM filter, there will be no noise enhancement problem. In addition, it does not have anything to do with the parameters $M$ and $K$ being even or odd. It turns out that the previous misconceptions in the literature that GFDM matrices can be singular or non-unitary are lying on the assumption that the prototype filter $\mathbf{g}$ is chosen to be raised cosine (RC) filters [4]. If we do not insist using RC filters (but use CMCM filters instead), the issues are automatically resolved. Furthermore, the expression in Eq. (2) also offers a low-complexity implementation of the precoding characterized by $\mathbf{A}$, as shown in Figure 2. The low-complexity implementation for the GFDM receivers can be shown to exist. But due to space limit, they are omitted here. Interested readers may refer to the elaborations in [2].

All the above findings suggest that choosing the GFDM prototype filter to be a CMCM filter (i.e., corresponding to a unitary matrix) has clear advantages in terms of both implementation
complexity and avoiding noise amplification. This is very different from most works in the GFDM literature which uses RC, root raised cosine (RRC), or other prototype filters [4]. The GFDM block size $S = MK$ can be chosen to be any composite number. On top of these advantages, the CMCM filter coefficients can be further designed to optimize relevant waveform performance metrics such as OOB and PAPR.

![Figure 2 Illustration of low-complexity implementation of GFDM precoding from Eq. (2)](image)

**Optimization of prototype filters**

In order to find the optimal coefficients for the GFDM matrix that achieves a minimum OOB, an optimization problem can be formulated as follows:

$$\begin{align*}
\text{minimize} & \quad \mathcal{S}_a(f, g) \\
\text{subject to} & \quad \mathbf{g} = \text{vec}(\mathbf{G} \mathbf{W}_M^H)/\sqrt{S} \\
& \quad \|\mathbf{g}\|_F = 1 \\
& \quad |[\mathbf{G}]_{k,m}| = 1, \ \forall k, m
\end{align*}$$

(3a)

(3b)

(3c)

(3d)

where $\mathcal{S}_a(f, g)$ is the power spectral density function of the transmitted signal when the prototype filter is chosen as $g$. The set $\mathcal{B}_O$ denotes the set of all out-of-band frequencies. Note that (3b) is equivalent to $\mathbf{G} = \sqrt{S} \text{reshape}(\mathbf{g}, K, M) \mathbf{W}_M$ and dictates the one-to-one correspondence of the prototype filter $\mathbf{g}$ and the characteristic matrix $\mathbf{G}$. Constraint (3c) is a power constraint and constraint (3d) enforces the GFDM matrix to be unitary.

While the objective function (3a) and the constraint (3b) are convex, constraints (3c) and (3d) are not, making the problem not be readily solved by existing convex optimization tools. However, techniques such as semidefinite relaxation can be applied and some iterative algorithms can be developed to obtain the optimal coefficients [3]. Figure 3(a) depicts the power spectral density of various GFDM waveforms. We observe that the OOB performance for OFDM is in general worse than GFDM. Among all GFDM waveforms with three different prototype filters, the RC filter is better than the Dirichlet filter which was known to corresponds to a unitary GFDM matrix. The “Optimized filter” according to Problem (3) has the best OOB performance (note that the dashed lines denote the boundaries of in-band and out-of-band frequencies). If we take a look at the symbol error rate (SER) performance at Figure 3(b), it is observed that the optimized filter does not have any SER loss compared to the OFDM case due to the no noise-enhancement constraint (in Eq. (3d)), while the RC filter suffers some SER loss.
Summary

In this letter, we briefly reviewed various types of OFDM-based waveforms for 5G NR and studied in particular the type based on a linear precoding matrix. Besides DFT-S-OFDM, perhaps the most popular precoding-based waveform, we found that GFDM matrices are worthy of being considered in the precoder design for this type of waveforms due to design flexibility and existence of low-complexity implementations. We studied and discovered some important properties of GFDM matrices that were less known in the literature which suggest that GFDM precoding can still be implemented efficiently, with virtually no performance loss, and possess a good OOBE property. In the future, many other performance metrics can be considered to obtain optimal coefficients for a precoding-based waveform using a GFDM matrix.


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